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Forked Communication Network Model with Non-Homogenous Bulk Arrivals and Phase Type Transmission

K. Srinivasa Rao[1], SK. Meeravali[2] & P. Srinivasa Rao[3]

Abstract

This paper introduces a forked communication network model with non-homogenous network model with bulk arrivals and phase type transmission. In this model it is assumed that the messages are converted into packets of random size and stored in buffers for forward transmission. The arrival of messages to the network follows Poisson process and the number of packets a message can be converted is random and follows a probability distribution. It is further assumed that the arrivals of packets are time dependent. As a result of it the arrival process follows a non-homogenous compound Poisson process. This type of scenario is visible at places like MAN, WAN and LAN. It is assumed that after completing transmission of the packet from the first node it may join either of the two buffers connected in tandem to the first node with certain probabilities or the packet may leave the network. It is also assumed that the transmission processes in three nodes follow Poisson processes. The transmission rate is adjusted on the content of the buffers connected to them. The performance of the network is analysed through obtaining the explicit expressions for the performance measures such as mean number of packets in the buffer, mean delay in transmission, throughput of the nodes, and variability of the content in the buffers. The sensitivity analysis of the model with respective to the changes in the parameters is also studied. It is observed that the performance measures are highly influenced by batch size distribution parameters. The dynamic band width allocation strategy can reduce the burstiness in buffers and mean delay in transmission. This model includes some of the earlier models as particular cases for specific values of the parameters.

Keywords: Non stationery network models, Phase type transmission, forked communication network, Performance measures, Non-homogenous compound Poisson process.

1. Introduction

In many practical situations arising at places like, computer communications, satellite and telecommunication data and voice transmission, ATM scheduling, network management, the communication network models play a dominant role. One of the major strings in developing the communication network models is replacing some of the assumptions in constituent processes of network where a more realistic practical consideration can be employed. In several communication network models the transmitter are nodes are connected in tandem Srinivasa Rao et.al [1]. In communication network models the arrival and service process are considered to be independent. But to have quality of service and to reduce burstiness in buffers one has to consider the transmission rate must depend on the number of packets in the buffer connected to it. Recently in literature some work has been reported with respect to dynamic bandwidth allocation (DBA) in which the transmission rate is linear dependent on content of to the buffer connected to it. Suresh Varma et.al [2], Padmavathiet.al [3], Ramasundari et.al[4], Srinivasa Rao et.al [5], have developed some communication network models with dynamic band width allocation. Sitha Rama Murthy et.al [6] have utilised the queueing with dynamic band width allocation for analysing the communication network. However, they assumed that the arrival of packets to the buffer are single and can be characterised by Poisson process. In some situations the arrival of packets cannot be characterised by Poisson process. In communication networks usually the messages arrived to the sources are converted into random number of packets based on the size of the message.

[1] Department of Statistics, College of Science & Technology, Andhra University, Visakhapatnam. E-mail: ksraoau@yahoo.co.in
[2] Department of CS&SE, College of Engg. (A), Andhra University, Visakhapatnam. E-mail:meerasha2002@gmail.com
[3] Department of CS&SE, College of Engg. (A), Andhra University, Visakhapatnam, E-mail: peri.srinivasarao@yahoo.com

This scenario is clearly visible in store and a forward communications network. That is the packets arrive to the buffer in bulk and can be characterised with compound Poisson process. The compound Poisson process also includes Poisson process as a particular case. Recently Nageswara Rao et.al [7], Srinivasa Rao et.al [8], Thirupati Rao et.al [9], Srinivasa Rao et.al [10], have developed and analysed some communication network models with bulk arrivals having dynamic bandwidth allocation. Hari Dass et.al [11] and Achutha Rao et.al [12], have used queueing models with bulk arrivals having load dependent service for analysing communication network models. In all these models they assumed that the arrivals are time dependent and characterised by compound Poisson process.

In many communication networks the arrivals of messages are time dependent. For instance the aggregate traffic in communication networks often bursty and remain unsmooth. Leland [13], established "the, actual traffic in Ethernet LAN exhibits the property of self-similarity and long range similarity". Rakesh Singhai [14] have analysed MAN and WAN traffic and established that the traffic exhibits time dependent arrivals. Crovella et.al [15], Murali Krishna [16], Feld Man et.al [17], have mentioned that TCP connection arrivals and inter arrival times of packets are non-homogenous. The time depended traffic in networks can besuitably characterised by non-homogenous Poisson process. William A Massey [18] has used the non-stationary Poisson process for analysing telecommunication models with time varying rates of arrivals. He also reviewed several works which supports time dependent behaviour of traffic flow models. Ward Width [19], reviewed the time dependent single server queues using diffusion approximations areken dells frame work. Suhasini et.al [20], [21], have developed parallel and series queueing model with non-homogenous bulk arrivals and applied it to the communication networks. However, no serious attempt is made to analyse forked communication network models with dynamic bandwidth allocation having non-stationary arrivals of traffic. Hence, in this paper we develop and analyse a forked communication network model with non-homogenous bulk arrivals having dynamic bandwidth allocation. Here it is assumed that the messages arrive from the sources converted into random number of packets and stored in buffers for forward transmission. The arrivals of packets to the first buffer are characterised by non-homogenous compound Poisson process. After completing the transmission from the first transmitter the packets may join either second or third buffer connected in tandem to the first node and parallel to each other with certain probability or may leave the network. Thistype of communication networks are called forked communication networks.

Using difference-differential equations the explicit expressions such as content of the buffers, throughput of the nodes, mean delay in transmission and utilization of the nodes are derived. The behaviour of the network model is analysed under transient conditions. The sensitivity of the model with respect to the changes of parameters on the performance measures is also discussed. A comparative study of the model with that of the homogenous bulk arrivals is discussed. Further work in this area of research is discussed in conclusions.

2. Communication Network Model

In this section, a forked communication network model with non-homogenous bulk arrivals having phase type transmission is discussed. Consider a communication network in which first node is connected to the second and third nodes in tandem and the message arrive to the first node are converted into a random number of packets and stored in first buffer connected to the first node. After transmitting from the first node, the packets may forwarded to second or third buffers which are connected in parallel for the forward transmission with the probabilities $\boldsymbol{\pi}$and $\mathbf{(1 - \pi - \delta)}$ where, the packet may be terminated after the first node with the probability $\boldsymbol{\delta}$. It is also assumed that the arrival of packets to the first buffer is in bulk size with random batch size having the probability mass function $\{C_k\}$. The transmission rate of each packet is adjusted before transmission depending on the content of the buffer connected to the transmitter.

Here, it is assumed that the arrival of packets follows non-homogenous compound Poisson process with the parameter $\lambda(t) = \alpha + \beta t$ and the number of transmissions at node 1, node 2 and node 3 follow Poisson processes with the parameters μ_1,μ_2 and μ_3. The queue discipline is First-In-First-Out (FIFO). The schematic diagram representing the communication network model is shown in Figure 1

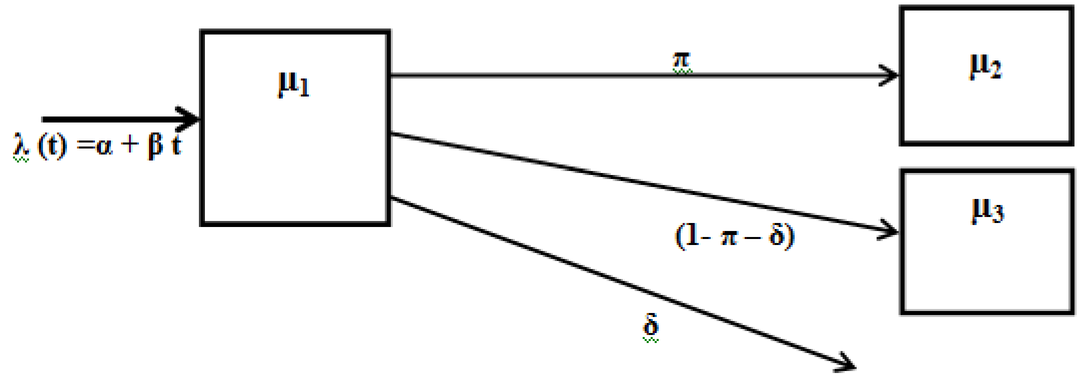

Fig 1 Schematic diagram of the queuing mode

Let $P_{n1,n2,n3}(t)$ be the probability that there are $\mathbf{n_1}$ packets in the first buffer, n2 packets in the second buffer and n3 packets in the third buffer at time t. the differential equations of the network are:

$$P^{1}_{n_1,n_2,n_3}(t) = -(\alpha+\beta(t)+n_1\mu_1+n_2\mu_2+n_3\mu_3+)P_{n_1,n_2,n_3}(t)+\lambda(t)\left[\sum_{k=1}^{n_1} c_k\, P_{n_1-k,n_2,n_3}(t)\right]+(n_1+1)\Pi\mu_1 P_{n_1+1,n_2-1,n_3}(t)+$$
$$(n_1+1)(1-\pi-\delta)\mu_1 P_{n_1+1,n_2,n_3-1}(t)+(n_2+1)\mu_2 P_{n_1+1,n_2+1,n_3}(t)+(n_3+1)\mu_3 P_{n_1,n_2,n_3+1}(t)$$
$$P^{1}_{0,n2,n3}(t) = -(\alpha+\beta(t)+n_2\mu_2+n_3\mu_3)P_{0,n2,n3}(t)+\Pi\mu_1 P_{1,n2-1,n3}(t)$$
$$+(1-\pi-\delta)\mu_1 P_{1,n2,n3-1}(t)+\delta\mu_1 P_{1,n2,n3}(t)+(n_2+1)\mu_2 P_{0,n2+1,n3}(t)+(n_3+1)\mu_3 P_{0,n2,n3+1}(t)$$
$$P^{1}_{n_1,0,n_3}(t) = -(\lambda(t)+n_1\mu_1+n_3\mu_3)P_{n_1,0,n_3}(t)+\lambda(t)\left[\sum_{k=1}^{n_1} c_k\, P_{n_1-k,0,n_3}(t)\right]+$$
$$(n_1+1)\delta\mu_1 P_{n_1+1,0,n_3}(t)+(n_1+1)(1-\pi-\delta)\mu_1 P_{n_1+1,0,n_3-1}(t)+\mu_2 P_{n_1,1,n_3}(t)+(n_3+1)\mu_3 P_{n_1,0,n_3+1}(t)$$

$$P^{1}_{n1,n2,0}(t) = -(\lambda(t)+n_1\mu_1+n_2\mu_2)P_{n1,n2,0}(t)+\lambda(t)\left[\sum_{k=1}^{n_1} c_k\, P_{n1-k,n2,0}(t)\right]+(n_1+1)\Pi\mu_1 P_{n+1,n2-1,0}(t)+$$
$$(n_1+1)\delta\mu_1 P_{n1+1,n2,0}(t)+(n_2+1)\mu_2 P_{n1,n2+1,0}(t)+\mu_3 P_{n1,n2,1}(t)$$
$$P^{1}_{0,0,n3}(t) = -(\lambda(t)+n_3\mu_3)P_{0,0,n3}(t)+(1-\pi-\delta)\mu_1 P_{1,0,n3-1}(t)+(n_1+1)\delta\mu_1 P_{1,0,n3}(t)+\mu_2 P_{0,1,n3}(t)+(n_3+1)\mu_3 P_{0,0,n3+1}(t)$$
$$P^{1}_{n1,0,0}(t) = -(\lambda(t)+n_1\mu_1)P_{n1,0,0}(t)+\lambda(t)\left[\sum_{k=1}^{n_1} c_k\, P_{n1-k,0,0}(t)\right]+(n+1)\delta\mu_1 P_{n1+1,0,0}(t)+\mu_2 P_{n1,1,0}(t)+\mu_3 P_{n1,0,1}(t)$$
$$P^{1}_{0,n2,0}(t) = -(\lambda(t)+n_2\mu_2)P_{0,n2,0}(t)+\pi\mu_1 P_{1,n2-1,0}(t)+(n_1+1)\delta\mu_1 P_{1,n2,0}(t)+(n_2+1)\mu_2 P_{0,n2+1,0}(t)+\mu_3 P_{0,n2,n1}(t)$$
$$P^{1}_{0,0,0}(t) = -\lambda(t)P_{0,0,0}(t)+\delta\mu_1 P_{1,0,0}(t)+\mu_2 P_{0,1,0}(t)+\mu_3 P_{0,0,1}(t)$$

(1)

The joint probability generating function $P_{n1,n2,n3}(t)$ is

$$p(z_{1,}z_{2,}z_{3,};t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty} z_1^{n_1} z_2^{n_2} z_3^{n_3}\; P_{n1,n2,n3}(t)$$

(2)

$$\frac{\partial p(z_1,z_2,z_3;t)}{\partial t} = \mu_1(\pi z_2+(\pi-\delta)z_3-z_1+\delta)\frac{\partial p(z_1,z_2,z_3;t)}{\partial z_1}+\mu_2(1-z_2)\frac{\partial p(z_1,z_2,z_3;t)}{\partial z_2}+$$

$$\mu_3(1-z_3)\frac{\partial p(z_1,z_2,z_3;t)}{\partial z_3}--\lambda(t)(1-c(z_1))\,p(z_1,z_2,z_3;t)$$

$$where, c(z_{1,}) = \sum_{k=1}^{\infty} z_1^k c_k \tag{3}$$

Solving the equation (3.2.3) by Lagrange's method the auxiliary equations are

$$\frac{dt}{1}=\frac{dz_1}{\mu_1\left(z_1-\pi z_2-(1-\pi-\delta)z_3-\delta\right)}=\frac{dz_2}{\mu_2(z_2-1)}=\frac{dz_3}{\mu_3(z_3-1)}=\frac{dp\left(z_{1,}z_{2,}z_{3,};t\right)}{\lambda(t)\left[c\left(z_1\right)-1\right]p\left(z_{1,}z_{2,}z_{3,};t\right)} \tag{4}$$

Solving the first and fourth terms in equation (4) we get

$$a=(z_3-1)e^{-\mu_3 t}$$

Solving the first and third terms in equation (4) we get

$$b=(z_2-1)e^{-\mu_2 t}$$

Solving the first and second terms in the equation (4) we get

$$c=\left[z_1-1+\frac{\pi\mu_1(z_2-1)}{\mu_2-\mu_1}+\frac{(1-\pi-\delta)\mu_1(z_3-1)}{\mu_3-\mu_1}\right]e^{-\mu_1 t}$$

Solving the first and fifth terms in the equation (4) we get

$$d=\exp\left\{-\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{i=0}^{r}\sum_{j=0}^{i} c_n\binom{n}{r}\binom{r}{i}\binom{i}{j}(-1)^i c^{r-i}\left(\frac{b\mu_1\pi}{\mu_2-\mu_1}\right)^{i-j}\left(\frac{a\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)^j\right.\right.$$
$$\left.\left.\left[\frac{\alpha}{\mu_1(r-i)+\mu_2(i-j)+\mu_3 j}-\frac{\beta}{\left[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j\right]^2}\right]\right\}\right\}$$

where a, b, c and d are arbitrary constants.
Using the initial conditions

$$P(z_1, z_2, z_{3;}t)$$
$$=\exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{i=0}^{r}\sum_{j=0}^{i} c_n\binom{n}{r}\binom{r}{i}\binom{i}{j}(-1)^i\left[(Z_1-1)+\left(\frac{(Z_2-1)\mu_1\pi}{\mu_2-\mu_1}\right)+\left(\frac{(Z_3-1)\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^{r-i}\right.$$
$$\left[\frac{(Z_2-1)\mu_1\pi}{\mu_2-\mu_1}\right]^{i-j}\left[\frac{(Z_3-1)\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right]^j$$
$$\left.\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\left[\left[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j\right]t\right]}\right]\left[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j\right]+\beta\left[e^{-\left[\left[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j\right]t\right]}-1\right]}{\left[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j\right]^2}\right]\right\} \tag{5}$$

3. Performance Measures of the System

In this section, the performance measures of the communication network under transient conditions are derived. The probability that the network is empty is obtained by expanding P (Z1, Z_2, $Z_{3;}$t) given in the equation (5) and collecting the constant terms as.

$$P_{0,0,0}(t) = \exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{i=0}^{r}\sum_{j=0}^{i} c_n \binom{n}{r}\binom{r}{i}\binom{i}{j}(-1)^{i+r}\left[1+\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right) + \left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^{r-i}\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)^{i-j}\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)^{j} \left[\frac{\left[\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j]t]}\right][\mu_1(r-i)+\mu_2(i-j)+\mu_3 j]+\beta\left[e^{-[[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j]t]}-1\right]\right]}{[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j]^2}\right]\right\} \quad (6)$$

We get the probability that the second buffer is empty as

$$P_{0,.,.}(t) = \exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n} c_n \binom{n}{r}(-1)^r\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-r\mu_1 t}\right](r\mu_1)+\beta\left[e^{-r\mu_1 t}-1\right]}{(r\mu_1)^2}\right]\right\} \quad (7)$$

The mean number of packets in the first buffer is

$$L_1(t) = \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{(\mu_1)^2}\right]\sum_{n=1}^{\infty} nc_n \quad (8)$$

The utilization of the first node is

$$U_1(t) = 1-\exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n} c_n \binom{n}{r}(-1)^r\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-r\mu_1 t}\right](r\mu_1)+\beta\left[e^{-r\mu_1 t}-1\right]}{(r\mu_1)^2}\right]\right\} \quad (9)$$

The throughput of the first node is

$$Thp_1(t) = \mu_1\left[1-\exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n} c_n \binom{n}{r}(-1)^r\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-r\mu_1 t}\right](r\mu_1)+\beta\left[e^{-r\mu_1 t}-1\right]}{(r\mu_1)^2}\right]\right\}\right] \quad (10)$$

The average delay in the first buffer is

$$W_1(t) = \left[\frac{\left[\frac{[(\alpha+\beta t)-\alpha e^{-\mu_1 t}](\mu_1)+\beta[e^{-\mu_1 t}-1]}{(\mu_1)^2}\right]\sum_{n=1}^{\infty} nc_n}{\mu_1\left[1-\exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n} c_n\binom{n}{r}(-1)^r\left[\frac{[(\alpha+\beta t)-\alpha e^{-r\mu_1 t}](r\mu_1)+\beta[e^{-r\mu_1 t}-1]}{(r\mu_1)^2}\right]\right\}\right]}\right] \quad (11)$$

The variance of the number of packets in the first buffer is

$$Var_1(t) = \sum_{n=1}^{\infty} nC_n\left[\left(\frac{n-1}{2}\right)\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-2\mu_1 t}\right](2\mu_1)+\beta\left[e^{-2\mu_1 t}-1\right]}{2\mu_1{}^2}\right] + \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{(\mu_1)^2}\right]\right] \quad (12)$$

The coefficient of variation of the number of packets in the first buffer is

$$CV_1(t) = \left[\frac{\sqrt{Var_1(t)}}{L_1(t)}\right].100$$

(13)

We get the probability that the second buffer is empty as (14)

$$P_{.,0,.}(t) = \exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{i=0}^{r} C_n \binom{n}{r}\binom{r}{i}(-1)^{i+r}\left[\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\right]^r \left[\frac{\left[\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-i)+\mu_2 i]t]}\right][\mu_1(r-i)+\mu_2 i]+\beta\left[e^{-[[\mu_1(r-i)+\mu_2 i]t]}-1\right]\right]}{[\mu_1(r-i)+\mu_2 i]^2}\right]\right\}$$

The mean number of packets in the second buffer is

$$L_2(t) = \left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\left[\left[\frac{[(\alpha+\beta t)-\alpha e^{-\mu_1 t}](\mu_1)+\beta[e^{-\mu_1 t}-1]}{(\mu_1)^2}\right] - \left[\frac{[(\alpha+\beta t)-\alpha e^{-\mu_2 t}](\mu_2)+\beta[e^{-\mu_2 t}-1]}{(\mu_2)^2}\right]\right].\sum_{n=1}^{\infty} nc_n$$

(15)

The utilization of the second node is

$$U_2(t) = 1-\exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{i=0}^{r} C_n \binom{n}{r}\binom{r}{i}(-1)^{i+r}\left[\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\right]^r \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-i)+\mu_2(i)]t]}[\mu_1(r-i)+\mu_2 i]+\beta\left[e^{-[[\mu_1(r-i)+\mu_2 i]t]}-1\right]\right]}{[\mu_1(r-i)+\mu_2 i]^2}\right]\right\}$$

(16)

The throughput of the second node is (17)

$$Thp_2(t) = \mu_2\left[1-\exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{i=0}^{r} C_n \binom{n}{r}\binom{r}{i}(-1)^{i+r}\left[\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\right]^r \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-i)+\mu_2(i)]t]}[\mu_1(r-i)+\mu_2 i]+\beta\left[e^{-[[\mu_1(r-i)+\mu_2 i]t]}-1\right]\right]}{[\mu_1(r-i)+\mu_2 i]^2}\right]\right\}\right]$$

The average delay in the second buffer is (18)

$$W_2(t) = \left[\frac{L_2(t)}{Thp_2(t)}\right]$$

$$= \left[\frac{\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\left[\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{(\mu_1)^2}\right]-\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_2 t}\right](\mu_2)+\beta\left[e^{-\mu_2 t}-1\right]}{(\mu_2)^2}\right]\right].\sum_{n=1}^{\infty} nc_n}{\mu_2\left[1-\exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{i=0}^{r} C_n\binom{n}{r}\binom{r}{i}(-1)^{i+r}\left[\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\right]^r\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-i)+\mu_2(i)]t]}\left[\mu_1(r-i)+\mu_2 i\right]+\beta\left[e^{-[[\mu_1(r-i)+\mu_2 i]t]}-1\right]\right]}{\left[\mu_1(r-i)+\mu_2 i\right]^2}\right]\right\}\right]}\right]$$

The variance of number of packets in the second node is

$$\mathrm{Var}_2(t) = \sum_{n=2}^{\infty} C_n\binom{n}{2}\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)^2\left[\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-2\mu_1 t}\right](2\mu_1)+\beta\left[e^{-2\mu_1 t}-1\right]}{2\mu_1{}^2}\right]\right.$$
$$-4\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-(\mu_1+\mu_2)t}\right](\mu_1+\mu_2)+\beta\left[e^{-(\mu_1+\mu_2)t}-1\right]}{(\mu_1+\mu_2)^2}\right]$$
$$\left.+\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-2\mu_2 t}\right](2\mu_2)+\beta\left[e^{-2\mu_2 t}-1\right]}{2\mu_2{}^2}\right]\right]$$
$$+\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\left[\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{\mu_1{}^2}\right]\right.$$
$$\left.-\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_2 t}\right](\mu_2)+\beta\left[e^{-\mu_2 t}-1\right]}{\mu_2{}^2}\right]\right].\sum_{n=1}^{\infty} nC_n \tag{19}$$

The coefficient of variation of the number of packets in the second buffer is

$$\mathrm{CV}_2(t) = \left[\frac{\sqrt{\mathrm{Var}_2(t)}}{L_2}\right].100 \tag{20}$$

We get the probability that the third buffer is empty as

$$P_{.,.,0}(t) = \exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{j=0}^{r} C_n\binom{n}{r}\binom{r}{j}(-1)^{r+j}\left[\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^r\right.$$
$$\left.\left[\frac{\left[\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-j)+\mu_3(j)]t]}\right]\left[\mu_1(r-j)+\mu_3 j\right]+\beta\left[e^{-[[\mu_1(r-j)+\mu_3 j]t]}-1\right]\right]}{\left[\mu_1(r-j)+\mu_3 j\right]^2}\right]\right\} \tag{21}$$

The mean number of packets in the third buffer is

$$L_3(t) = \left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\left[\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{(\mu_1)^2}\right]\right.$$
$$\left.-\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_3 t}\right](\mu_3)+\beta\left[e^{-\mu_3 t}-1\right]}{(\mu_3)^2}\right]\right].\sum_{n=1}^{\infty} nc_n \tag{22}$$

The utilization of the third node is

$$U_3(t) = 1 - \exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{j=0}^{r} C_n \binom{n}{r}\binom{r}{j}(-1)^{r+j}\left[\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^r \left[\frac{\left[\left[(\alpha+\beta t) - \alpha e^{-[[\mu_1(r-j)+\mu_3(j)]t]}\right][\mu_1(r-j)+\mu_3 j] + \beta\left[e^{-[[\mu_1(r-j)+\mu_3 j]t]} - 1\right]\right]}{[\mu_1(r-j)+\mu_3 j]^2}\right]\right\} \tag{23}$$

The throughput of the third node is

$$Thp_3(t) = \mu_3.\left[1 - \exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{j=0}^{r} C_n \binom{n}{r}\binom{r}{j}(-1)^{r+j}\left[\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^r \left[\frac{\left[\left[(\alpha+\beta t) - \alpha e^{-[[\mu_1(r-j)+\mu_3(j)]t]}\right][\mu_1(r-j)+\mu_3 j] + \beta\left[e^{-[[\mu_1(r-j)+\mu_3 j]t]} - 1\right]\right]}{[\mu_1(r-j)+\mu_3 j]^2}\right]\right\}\right] \tag{24}$$

The average delay in the third buffer is

$$W_3(t) = \left[\frac{\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\left[\left[\frac{[(\alpha+\beta t)-\alpha e^{-\mu_1 t}](\mu_1)+\beta[e^{-\mu_1 t}-1]}{(\mu_1)^2}\right] - \left[\frac{[(\alpha+\beta t)-\alpha e^{-\mu_3 t}](\mu_3)+\beta[e^{-\mu_3 t}-1]}{(\mu_3)^2}\right]\right].\sum_{n=1}^{\infty} nc_n}{\mu_3.\left[1-\exp\left\{\sum_{n=1}^{\infty}\sum_{r=1}^{n}\sum_{j=0}^{r} C_n\binom{n}{r}\binom{r}{j}(-1)^{r+j}\left[\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^r\left[\frac{\left[\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-j)+\mu_3(j)]t]}\right][\mu_1(r-j)+\mu_3 j]+\beta\left[e^{-[[\mu_1(r-j)+\mu_3 j]t]}-1\right]\right]}{[\mu_1(r-j)+\mu_3 j]^2}\right]\right\}\right]}\right] \tag{25}$$

The variance of number of packets in the second node is

$$Var_3(t) = E[N_3{}^2 - N_3] + E[N_3] - (E[N_2])^2$$

$$= \sum_{n=2}^{\infty} C_n \binom{n}{2} \left(\frac{\mu_1\theta}{\mu_3 - \mu_1}\right)^2 \left[\left[\frac{[(\alpha+\beta t) - \alpha e^{-2\mu_1 t}](2\mu_1) + \beta[e^{-2\mu_1 t} - 1]}{2{\mu_1}^2}\right] - 4\left[\frac{\left[(\alpha+\beta t) - \alpha e^{-(\mu_1+\mu_3)t}\right](\mu_1+\mu_3) + \beta\left[e^{-(\mu_1+\mu_3)t} - 1\right]}{(\mu_1+\mu_3)^2}\right] + \left[\frac{[(\alpha+\beta t) - \alpha e^{-2\mu_3 t}](2\mu_3) + \beta[e^{-2\mu_3 t} - 1]}{2{\mu_3}^2}\right]\right] + \left(\frac{\mu_1\theta}{\mu_3 - \mu_1}\right)\left[\left[\frac{[(\alpha+\beta t) - \alpha e^{-\mu_1 t}](\mu_1) + \beta[e^{-\mu_1 t} - 1]}{{\mu_1}^2}\right] - \left[\frac{[(\alpha+\beta t) - \alpha e^{-\mu_3 t}](\mu_3) + \beta[e^{-\mu_3 t} - 1]}{{\mu_3}^2}\right]\right] . \sum_{n=1}^{\infty} nC_n \tag{26}$$

The coefficient of variation of the number of packets in the third buffer is

$$CV_3(t) = \left[\frac{\sqrt{Var_3(t)}}{L_3(t)}\right].100 \tag{27}$$

4 Performance Measures of the System when the Batch Size Is Uniform

The performance of the communication network is highly influenced by the structure of the batch size distribution. In most of the communication systems the number of packets that a message can be converted is random and follows a uniform (rectangular) distribution with parameters a & b. The probability mass function of the number of packets in a message is

$$C_k = \frac{1}{(b-a+1)} \text{ for } k = a, a+1, \ldots\ldots, b.$$

The mean number of packets in a message is $\frac{(a+b)}{2}$ and its variance is $\frac{1}{12}[b-a+1]^2$.

Substituting the values of C_k in equation (5), we get the joint probability generating function of the number of packets in both the buffers.

Probability that the network is empty is:

$$P_{0,0,0}(t) = \exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\sum_{i=0}^{r}\sum_{j=0}^{i}\left[\frac{1}{b-a+1}\right]\binom{n}{r}\binom{r}{i}\binom{i}{j}(-1)^{i+r}\left[1 + \left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right) + \left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^{r-i}\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)^{i-j}\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)^{j} \left[\frac{\left[\left[(\alpha+\beta t) - \alpha e^{-[[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j]t]}\right][\mu_1(r-i)+\mu_2(i-j)+\mu_3 j] + \beta\left[e^{-[[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j]t]} - 1\right]\right]}{[\mu_1(r-i)+\mu_2(i-j)+\mu_3 j]^2}\right]\right\} \tag{28}$$

We get the probability that the second buffer is empty as

$$P_{0,.,.}(t) = \exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\left(\frac{1}{b-a+1}\right)\binom{n}{r}(-1)^r\left[\frac{[(\alpha+\beta t) - \alpha e^{-r\mu_1 t}](r\mu_1) + \beta[e^{-r\mu_1 t} - 1]}{(r\mu_1)^2}\right]\right\} \tag{29}$$

The mean number of packets in the first buffer is (30)

$$L_1(t)=\left[\frac{a+b}{2{\mu_1}^2}\right]\left[\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]\right]$$

The utilization of the first node is

$$U_1(t)=1-\exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\left(\frac{1}{b-a+1}\right)\binom{n}{r}(-1)^r\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-r\mu_1 t}\right](r\mu_1)+\beta\left[e^{-r\mu_1 t}-1\right]}{(r\mu_1)^2}\right]\right\}$$

(31)

The throughput of the first node is

$$Thp_1(t)=\mu_1\left[1-\exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\left(\frac{1}{b-a+1}\right)\binom{n}{r}(-1)^r\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-r\mu_1 t}\right](r\mu_1)+\beta\left[e^{-r\mu_1 t}-1\right]}{(r\mu_1)^2}\right]\right\}\right]$$

(32)

The average delay in the first buffer is

$$W_1(t)=\left[\frac{L_1(t)}{Thp_1(t)}\right]$$

$$=\left[\frac{\left[\frac{a+b}{2{\mu_1}^2}\right]\left[\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]\right]}{\mu_1\left[1-\exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\left(\frac{1}{b-a+1}\right)\binom{n}{r}(-1)^r\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-r\mu_1 t}\right](r\mu_1)+\beta\left[e^{-r\mu_1 t}-1\right]}{(r\mu_1)^2}\right]\right\}\right]}\right]$$

(33)

The variance of the number of packets in the first buffer is

$$Var_1(t)=\sum_{n=a}^{b}\left(\frac{1}{b-a+1}\right)n\left\{\left(\frac{n-1}{2}\right)\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-2\mu_1 t}\right](2\mu_1)+\beta\left[e^{-2\mu_1 t}-1\right]}{2{\mu_1}^2}\right]+\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{(\mu_1)^2}\right]\right\}$$

(34)

The coefficient of variation of the number of packets in the first buffer is (35)

$$CV_1(t)=\left[\frac{\sqrt{Var_1(t)}}{L_1(t)}\right].100$$

We get the probability that the second buffer is empty as

$$P_{.,0,.}(t)=\exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\sum_{i=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{n}{r}\binom{r}{i}(-1)^{i+r}\left[\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\right]^r\left[\frac{\left[\left[(\alpha+\beta t)-\alpha e^{-\left[[\mu_1(r-i)+\mu_2 i]t\right]}\right]\left[\mu_1(r-i)+\mu_2(i)\right]+\beta\left[e^{-\left[[\mu_1(r-i)+\mu_2 i]t\right]}-1\right]\right]}{\left[\mu_1(r-i)+\mu_2 i\right]^2}\right]\right\}$$

(36)

The mean number of packets in the second buffer is

$$L_2(t)=\left(\frac{\mu_1\pi(a+b)}{2(\mu_2-\mu_1)}\right)\left\{\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{(\mu_1)^2}\right]-\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_2 t}\right](\mu_2)+\beta\left[e^{-\mu_2 t}-1\right]}{(\mu_2)^2}\right]\right\}$$

(37)

The utilization of the second node is

$$U_2(t) = 1 - \exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\sum_{i=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{n}{r}\binom{r}{i}(-1)^{i+r}\left[\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\right]^r \left[\frac{\left[\left[(\alpha+\beta t) - \alpha e^{-\left[\left[\mu_1(r-i)+\mu_2 i\right]t\right]}\right]\left[\mu_1(r-i)+\mu_2(i)\right] + \beta\left[e^{-\left[\left[\mu_1(r-i)+\mu_2 i\right]t\right]} - 1\right]\right]}{\left[\mu_1(r-i)+\mu_2 i\right]^2}\right]\right\} \tag{38}$$

The throughput of the second node is

$$Thp_2(t) = \mu_2\left[1 - \exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\sum_{i=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{n}{r}\binom{r}{i}(-1)^{i+r}\left[\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\right]^r \left[\frac{\left[\left[(\alpha+\beta t) - \alpha e^{-\left[\left[\mu_1(r-i)+\mu_2 i\right]t\right]}\right]\left[\mu_1(r-i)+\mu_2(i)\right] + \beta\left[e^{-\left[\left[\mu_1(r-i)+\mu_2 i\right]t\right]} - 1\right]\right]}{\left[\mu_1(r-i)+\mu_2 i\right]^2}\right]\right\}\right] \tag{39}$$

The average delay in the second buffer is

$$W_2(t) = \left[\frac{\left(\frac{\mu_1\pi(a+b)}{2(\mu_2-\mu_1)}\right)\left\{\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{(\mu_1)^2}\right] - \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_2 t}\right](\mu_2)+\beta\left[e^{-\mu_2 t}-1\right]}{(\mu_2)^2}\right]\right\}}{\mu_2\left[1-\exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\sum_{i=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{n}{r}\binom{r}{i}(-1)^{i+r}\left[\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)\right]^r\left[\frac{\left[\left[(\alpha+\beta t)-\alpha e^{-\left[\left[\mu_1(r-i)+\mu_2 i\right]t\right]}\right]\left[\mu_1(r-i)+\mu_2(i)\right]+\beta\left[e^{-\left[\left[\mu_1(r-i)+\mu_2 i\right]t\right]}-1\right]\right]}{\left[\mu_1(r-i)+\mu_2 i\right]^2}\right]\right\}\right]}\right] \tag{40}$$

The variance of number of packets in the second node is

$$\begin{aligned}Var_2(t) = \sum_{n=a}^{b}\binom{n}{2}\left[\frac{1}{b-a+1}\right]\left(\frac{\mu_1\pi}{\mu_2-\mu_1}\right)^2 &\left[\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-2\mu_1 t}\right](2\mu_1)+\beta\left[e^{-2\mu_1 t}-1\right]}{2\mu_1{}^2}\right]\right. \\ &- 4\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-(\mu_1+\mu_2)t}\right](\mu_1+\mu_2)+\beta\left[e^{-(\mu_1+\mu_2)t}-1\right]}{(\mu_1+\mu_2)^2}\right] \\ &\left.+ \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-2\mu_2 t}\right](2\mu_2)+\beta\left[e^{-2\mu_2 t}-1\right]}{2\mu_2{}^2}\right]\right] \\ &+ \frac{\mu_1\pi(a+b)}{2(\mu_2-\mu_1)}\left[\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{(\mu_1)^2}\right]\right. \\ &\left.- \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_2 t}\right](\mu_2)+\beta\left[e^{-\mu_2 t}-1\right]}{(\mu_2)^2}\right]\right]\end{aligned}$$

(41)

The coefficient of variation of the number of packets in the second buffer is

$$CV_2(t) = \left[\frac{\sqrt{Var_2(t)}}{L_2(t)}\right].100$$

(42)

The probability that the third buffer is empty is

$$P_{.,.,0}(t) = \exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\sum_{j=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{n}{r}\binom{r}{j}(-1)^{r+j}\left[\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^r \left[\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-j)+\mu_3 j]t]}\right][\mu_1(r-j)+\mu_3(j)]+\beta\left[e^{-[[\mu_1(r-j)+\mu_3 j]t]}-1\right]}{[\mu_1(r-j)+\mu_3 j]^2}\right]\right]\right\}$$

(43)

The mean number of packets in the third buffer is

$$L_3(t) = \left(\frac{\mu_1(1-\pi-\delta)(a+b)}{2(\mu_3-\mu_1)}\right)\left[\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_1 t}\right](\mu_1)+\beta\left[e^{-\mu_1 t}-1\right]}{(\mu_1)^2}\right] - \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-\mu_3 t}\right](\mu_3)+\beta\left[e^{-\mu_3 t}-1\right]}{(\mu_3)^2}\right]\right]$$

(44)

The utilization of the third node is

$$U_3(t) = 1-\exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\sum_{j=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{n}{r}\binom{r}{j}(-1)^{r+j}\left[\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^r \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-j)+\mu_3 j]t]}\right][\mu_1(r-j)+\mu_3(j)]+\beta\left[e^{-[[\mu_1(r-j)+\mu_3 j]t]}-1\right]}{[\mu_1(r-j)+\mu_3 j]^2}\right]\right\}$$

(45)

The throughput of the third node is

$$Thp_3(t) = \mu_3.\left[1-\exp\left\{\sum_{n=a}^{b}\sum_{r=1}^{a}\sum_{j=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{n}{r}\binom{r}{j}(-1)^{r+j}\left[\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^r \left[\frac{\left[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-j)+\mu_3 j]t]}\right][\mu_1(r-j)+\mu_3(j)]+\beta\left[e^{-[[\mu_1(r-j)+\mu_3 j]t]}-1\right]}{[\mu_1(r-j)+\mu_3 j]^2}\right]\right\}\right]$$

(46)

The average delay in the third buffer is

$$W_3(t) = \left[\frac{L_3(t)}{Thp_3(t)}\right] \quad (47)$$

$$= \frac{\left(\frac{\mu_1(1-\pi-\delta)(a+b)}{2(\mu_3-\mu_1)}\right)\left[\left[\frac{[(\alpha+\beta t)-\alpha e^{-\mu_1 t}](\mu_1)+\beta[e^{-\mu_1 t}-1]}{(\mu_1)^2}\right]-\left[\frac{[(\alpha+\beta t)-\alpha e^{-\mu_3 t}](\mu_3)+\beta[e^{-\mu_3 t}-1]}{(\mu_3)^2}\right]\right]}{\mu_3.\Big[1-\exp\Big\{\sum_{n=a}^{b}\sum_{r=1}^{a}\sum_{j=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{n}{r}\binom{r}{j}(-1)^{r+j}\left[\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)\right]^r \Big[\frac{\big[[(\alpha+\beta t)-\alpha e^{-[[\mu_1(r-j)+\mu_3 j]t]}\big][\mu_1(r-j)+\mu_3(j)]+\beta\big[e^{-[[\mu_1(r-j)+\mu_3 j]t}}{[\mu_1(r-j)+\mu_3 j]^2}}$$

The variance of number of packets in the second node is

$$\begin{aligned}\mathrm{Var}_3(t) = \sum_{n=a}^{b}\binom{n}{2}\left[\frac{1}{b-a+1}\right]\left(\frac{\mu_1(1-\pi-\delta)}{\mu_3-\mu_1}\right)^2 \Bigg[&\left[\frac{[(\alpha+\beta t)-\alpha e^{-2\mu_1 t}](2\mu_1)+\beta[e^{-2\mu_1 t}-1]}{2\mu_1^{\,2}}\right] \\ &- 4\left[\frac{\left[(\alpha+\beta t)-\alpha e^{-(\mu_1+\mu_3)t}\right](\mu_1+\mu_3)+\beta\left[e^{-(\mu_1+\mu_3)t}-1\right]}{\left((\mu_1+\mu_3)\right)^2}\right] \\ &+ \left[\frac{[(\alpha+\beta t)-\alpha e^{-2\mu_3 t}](2\mu_3)+\beta[e^{-2\mu_3 t}-1]}{2\mu_3^{\,2}}\right]\Bigg] \\ &+ \frac{\mu_1(1-\pi-\delta)(a+b)}{2(\mu_3-\mu_1)}\Bigg[\left[\frac{[(\alpha+\beta t)-\alpha e^{-\mu_1 t}](\mu_1)+\beta[e^{-\mu_1 t}-1]}{(\mu_1)^2}\right] \\ &- \left[\frac{[(\alpha+\beta t)-\alpha e^{-\mu_3 t}](\mu_3)+\beta[e^{-\mu_3 t}-1]}{(\mu_3)^2}\right]\Bigg]\end{aligned}$$

(48)

The coefficient of variation of the number of packets in the third buffer is

$$CV_3(t) = \left[\frac{\sqrt{\mathrm{Var}_3(t)}}{L_3(t)}\right].100$$

(49)

4.1 Performance Measures of the Communication Network Model with Uniform Batch Size Distribution

The performance of the forked communication network model is discussed through a numerical illustration. Different values of the model parameters are considered for bandwidth allocation, arrival of packets, transmission rates at nodes and probabilities of arrival of packets at node 2 and node 3. After interacting with Internet service providers (ISPs), it is considered that the message arrival rate (α, β) varies form 1×10^4 messages/second to 5×10^4 messages/second. The number of packets that can be converted into a message varies from message to message depending on the length of the message. The numbers of arrivals of packets to the buffer are in batches of random size. The batch size is assumed to follow uniform distribution with parameters (a, b). The transmission rate (μ_1) varies from 3×10^4 packets/second to 6×10^4 packets/second. The transmission rate (μ_2) which varies from 8×10^4 packets/second to 13×10^4 packets/second. The transmission rate (μ_3) which varies from 8×10^4 to 25×10^4 packets/second. The transmission rate of each node depends on the number of packets in the buffer connected to it at that instant.

The transient behaviour of the network model is studied through computing the performance measure with respect to change in time. The following set of values for the model parameters are considered for analysis.

t = 0.2, 0.5, 0.8, 1.2, 2.0 seconds, a=1, 2, 3, 4, 5, b=10, 15, 20, 25, 30

α=3.0, 3.5, 4.0, 4.5, 5.0 (with multiplication of 10^4 packets/second).
β=1.5, 2.0, 2.5, 3.5, 4.0(with multiplication of 10^4 packets/second).
μ_1=3, 4, 5, 6 (with multiplication of 10^4 packets/second).
μ_2=8, 9, 10, 11, 12 (with multiplication of 10^4 packets/second).
μ_3=8, 12, 16, 20, 25 (with multiplication of 10^4 packets/second).
δ= 0.1, 0.2, 0.3, 0.4, 0.5. and π=0.3.

Table 1: Values of Network and Buffer Emptiness

t*	a	b	$\alpha^{\#}$	$\beta^{\#}$	$\mu_1^{\$}$	$\mu_2^{\$}$	$\mu_3^{\$}$	π	(1-π-δ)	δ	$P_{0,0,0}(t)$	$P_{0,.,.}(t)$	$P_{.,0,.}(t)$	$P_{.,.,0}(t)$
0.2	5	25	4	2	4	10	20	0.3	0.3	0.4	0.4318	0.4321	0.657	0.7285
0.5	5	25	4	2	4	10	20	0.3	0.3	0.4	0.11	0.1136	0.3391	0.5052
0.8	5	25	4	2	4	10	20	0.3	0.3	0.4	0.0321	0.0367	0.2259	0.4139
1.2	5	25	4	2	4	10	20	0.3	0.3	0.4	0.0105	0.0136	0.16	0.3456
2.0	5	25	4	2	4	10	20	0.3	0.3	0.4	0.0023	0.0034	0.0926	0.2547
0.5	**1**	25	4	2	4	10	20	0.3	0.3	0.4	0.1208	0.127	0.3872	0.5512
0.5	**2**	25	4	2	4	10	20	0.3	0.3	0.4	0.1161	0.1215	0.3739	0.5389
0.5	**3**	25	4	2	4	10	20	0.3	0.3	0.4	0.1133	0.1188	0.3614	0.5272
0.5	**4**	25	4	2	4	10	20	0.3	0.3	0.4	0.1114	0.1155	0.3499	0.516
0.5	5	**10**	4	2	4	10	20	0.3	0.3	0.4	0.1189	0.1278	0.5131	0.6786
0.5	5	**15**	4	2	4	10	20	0.3	0.3	0.4	0.1139	0.1201	0.4369	0.6094
0.5	5	**20**	4	2	4	10	20	0.3	0.3	0.4	0.1115	0.116	0.3811	0.5525
0.5	5	**25**	4	2	4	10	20	0.3	0.3	0.4	0.11	0.1136	0.3391	0.5052
0.5	5	**30**	4	2	4	10	20	0.3	0.3	0.4	0.1092	0.1121	0.3066	0.4655
0.5	5	25	**3.0**	2	4	10	20	0.3	0.3	0.4	0.1796	0.184	0.4308	0.5866
0.5	5	25	**3.5**	2	4	10	20	0.3	0.3	0.4	0.1406	0.1446	0.3822	0.5444
0.5	5	25	**4.5**	2	4	10	20	0.3	0.3	0.4	0.0862	0.0893	0.3008	0.4688
0.5	5	25	**5.0**	2	4	10	20	0.3	0.3	0.4	0.0674	0.0702	0.2669	0.4351
0.5	5	25	4	**1.5**	4	10	20	0.3	0.3	0.4	0.1171	0.1209	0.3497	0.5161
0.5	5	25	4	**2.5**	4	10	20	0.3	0.3	0.4	0.1034	0.1068	0.3287	0.4946
0.5	5	25	4	**3.0**	4	10	20	0.3	0.3	0.4	0.0972	0.1004	0.3187	0.4842
0.5	5	25	4	**3.5**	4	10	20	0.3	0.3	0.4	0.0913	0.0944	0.309	0.474
0.5	5	25	4	2	**3**	10	20	0.3	0.3	0.4	0.107	0.1082	0.3707	0.5364
0.5	5	25	4	2	**4**	10	20	0.3	0.3	0.4	0.1100	0.1160	0.3391	0.5052
0.5	5	25	4	2	**5**	10	20	0.3	0.3	0.4	0.1154	0.1235	0.3224	0.4891
0.5	5	25	4	2	**6**	10	20	0.3	0.3	0.4	0.1234	0.1385	0.3138	0.4816
0.5	5	25	4	2	4	**8**	20	0.3	0.3	0.4	0.1095	0.1136	0.2972	0.5052
0.5	5	25	4	2	4	**9**	20	0.3	0.3	0.4	0.1098	0.1136	0.3184	0.5052
0.5	5	25	4	2	4	**11**	20	0.3	0.3	0.4	0.1103	0.1136	0.3592	0.5052
0.5	5	25	4	2	4	**12**	20	0.3	0.3	0.4	0.1104	0.1136	0.3785	0.5052
0.5	5	25	4	2	4	10	**8**	0.3	0.3	0.4	0.1087	0.1136	0.3391	0.2972
0.5	5	25	4	2	4	10	**12**	0.3	0.3	0.4	0.1094	0.1136	0.3391	0.3785
0.5	5	25	4	2	4	10	**16**	0.3	0.3	0.4	0.1098	0.1136	0.3391	0.448
0.5	5	25	4	2	4	10	**22**	0.3	0.3	0.4	0.1101	0.1136	0.3391	0.5299
0.5	5	25	4	2	4	10	**25**	0.3	0.3	0.4	0.1102	0.1136	0.3391	0.563
0.5	5	25	4	2	4	10	20	**0.3**	**0.6**	**0.1**	0.1094	0.1136	0.3391	0.3267
0.5	5	25	4	2	4	10	20	**0.3**	**0.5**	**0.2**	0.1096	0.1136	0.3391	0.3703
0.5	5	25	4	2	4	10	20	**0.3**	**0.4**	**0.3**	0.1098	0.1136	0.3391	0.4277
0.5	5	25	4	2	4	10	20	**0.3**	**0.3**	**0.4**	0.11	0.1136	0.3391	0.5052
0.5	5	25	4	2	4	10	20	**0.3**	**0.2**	**0.5**	0.1103	0.1136	0.3391	0.613

***= seconds, #= Multiple of 10,000 messages/second, $= Multiple of 10,000 packets/second**

From the equations (28), (29), (36) and (43), the probability of network emptiness and emptiness of buffers emptiness are computedfor different values of t, a, b, α, β, μ_1, μ_2, μ_3, π and δ and given in Table 1. It is observed that the probability of emptiness of the communication network and the three buffers are highly sensitive with respect to change in time.

As time (t) varies from 0.2 to 2.0 seconds, the probability of emptiness in the network reduces from 0.4318 to 0.0023, when other parameters are fixed. Similarly, the probabilities of emptiness of three buffers reduce from 0.4321 to 0.0034, 0.657 to 0.0926 and 0.7285 to 0.2547 for node 1, node 2 and node 3 respectively. The decrease in node 1 is more rapid when it is compared to node2 and node 3.

When the batch size distribution parameter (a) varies from 1 to 4, the probability of emptiness of the network decreases from 0.1208 to 0.1114 when other parameters are fixed. The same phenomenon is observed with respect to the first, second, and third buffers. The probability of emptiness decrease from 0.127 to 0.1155, 0.3872 to 0.3499 and 0.5512 to 0.516 respectively, for the first, second and third buffers.

When batch size distribution parameter (b) varies from 10 to 30, the probability of emptiness of the whole network decreases from 0.1189 to 0.1092 when other parameters are fixed. The same phenomenon is observed with respect to the first, second and third nodes. The probability of emptiness of the first, second and third buffers decrease from 0.1278 to 0.1121, 0.5131 to 0.3066 and 0.6786 to 0.4655 respectively.

The influence of arrival rate of messages on the emptiness at node 1, node 2 and node 3 is also studied. As the arrival rate parameters α and β varies from $3.0\text{x}10^4$ packets/second to $5.0\text{x}10^4$ packets/second and $1.5\text{x}10^4$ packets/second to $3.5\text{x}10^4$ packets/second respectively, the probability of emptiness of the network decreases. The same phenomenon is observed at node 1, node 2 and node 3 when the other parameters are fixed It is observed that the decrease in emptiness is more rapid in node 1 when compared to node 2 and node 3.

When the transmission rate of node 1(μ_1) varies from $3\text{x}10^4$ packets/sec to $6 \text{ x } 10^4$ packets/sec, the probability of emptiness of the network and the first buffer increase from 0.107 to 0.1234 and 0.1082 to 0.1385 respectively and the probability of emptiness of the second and third buffers decrease from 0.3707 to 0.3138 and 0.5364 to 0.4816 when other parameters remain fixed. When the transmission rate of node 2(μ_2) varies from $8 \text{ x}10^4$ packets/sec to $12 \text{ x } 10^4$ packets/sec, the probability of emptiness of the network and the second buffer increase from 0.1095 to 0.1104 and 0.2972 to 0.3785 respectively when other parameters remain fixed. Similarly the transmission rate of node 3(μ_3) varies from $8 \text{ x}10^4$ packets/sec to $22 \text{ x } 10^4$ packets/sec, the probability of emptiness of the network and the third buffer increase from 0.1087 to 0.1102 and 0.2972 to 0.5299 when other parameters remain fixed.

When the leaving rate δ varies from 0.1 to 0.5 then the probability of emptiness of the network and the third buffer increase from 0.1094 to 0.1103 and 0.3267 to 0.613 respectively, but for the first and second buffers remains constant at 0.1136 and 0.3391, when other parameters remain fixed.

From the equations (30), (37), (44), (31), (38) and (45), the mean number of packets and the utilization of the network for individual nodes are computed for different values of t, a, b, α, β, μ_1, μ_2, μ_3, π, δ and are given Table 2.

It observed that after 0.2 seconds, the first buffer is having on an average of 8726 packets, after 0.5 seconds it is rapidly raised to an average of 150,987 packets. After 0.8 second, the first buffer is containing 185,600 packets and there after system stabilizes and the average number of packets remains constant for fixed values of other parameters. It also observed that as time (t) varies from 0.2 to 2.0 seconds, average content in the second buffer, third buffers and the network increase from 6398 to 32841 packets, 4167 to 16647 packets, and from 97481 to 330694 packets respectively.

As the batch size distribution parameter (a) varies from 1 to 5, the first buffer, second buffer, third buffer and the network average content increases from 130856 packets to 150987 packets, 13758 packets to 15874 packets, 7426 packets to 8568 packets, and from 152039 packets to 175429 packets respectively when other parameters are fixed. As the batch size distribution parameter (b) varies from10 to 30 , the first buffer , second buffer, third buffer and average content increase from 75494 packets to 176152 packets, 7937 packets to 18520 packets, 4284 packets to 9996 packets, and form 87715 packets to 204668 packets respectively when other parameters are fixed.

As the arrival rate parameter (α) varies from $3.0\text{x}10^4$ messages/second to $5.0\text{x}10^4$ messages/second, the first buffer, second buffer, third buffer and average content in the buffers increase from 11856 packets to 183412 packets, 12369 packets to 19379 packets, 6698 packets to 8295 packets and 13763 packets to 213229 packets for first, second and third buffers respectively when other parameters are fixed.

As the arrival rate parameters (β) varies from 1.5×10^4 messages/second to 3.5×10^4 messages/second average content in the buffers increase 145665 packets to 166973 packets, 15411 packets to 17264 packets, 8295 packets to 9386 packets and 169372 packets to 19360 packets first, second and third buffers respectively when other parameters are fixed.

As the transmission rate of node 1 (μ_1) varies form 3.0×10^4 packets/second to 6.0×10^4 packets /second, average content of the first buffer and the network decrease form 179478 packets to 112103 packets and from 20903 packets to 14001 packets respectively, the average contents of the second and third buffers increase from 13868 packets to 18219 packets and 7756 packets to 9688 packets respectively when other parameters remain fixed.

As the transmission rate of node 2 (μ_2) varies form 8.0×10^4 packets/second to 12.0×10^4 packets /second, the average content of the second buffer and the network decrease form 18964 packets to 13603 packets and from 20903 packets to 173519 packets respectively, the average contents of the first and third buffers remain constant at 150987 packets and 8568respectively, when other parameters remain fixed.

As the transmission of node 3 (μ_3) varies form 8.0×10^4 packets/second to 22.0×10^4 packets /second, the average contents of the third buffer and network decrease form 18964 packets to 7834 packets and from 185825 packets to 173802 packets respectively, the average content of the first and second buffers remain constant at 150987 packets and 15874 when other parameters remain fixed.

When the leaving rate parameter (δ) varies from 0.1 to 0.5, the average content of the third buffer and the network decrease from 17136 packets to 5712 packets and 183997 packets to 172573 packets respectively and the average content of the first and second buffer remain constant when other parameters remain fixed.

As the time (t) varies from 0.2 to 2.0 seconds, the utilization of the first, second and third nodes increases from 5679 packets to 9966 packets, 3430 packets to 9074 packets and 2715 packets to 7433 packets, when other parameters are remain fixed.

As the batch size distribution parameter (a) varies from 1 to 5, the utilization of the first, second and third nodes increase from 8730 packets to 8845 packets, 6128 packets to 6501 packets and 4488 packets to 4840 packets respectively when other parameters are fixed. As the batch size distribution parameter (b) increase varies from 10 to 30, the utilization of three nodes increase from 8722 packets to 8879 packets, 4869 packets to 6934 packets and 3214 packets to 5345 packets when other parameters remain fixed .

As the arrival rate parameter (α) varies form 3.0×10^4 to 5.0×10^4 messages/ second, the utilization of the three nodes increase from 8160 packets to 9298 packets, 5692 packets to 7331 packets and 4134 packets to 5649 packets respectively when other parameters are fixed. As the arrival rate parameter (β) varies form 1.5×10^4 to 3.5×10^4 messages/ second, the utilization of the three nodes increase from 8751 packets to 9056 packets, 6503 packets to 6910 packets and 4839 packets to 5260 packets respectively when other parameters are fixed When the leaving parameter (δ) varies from 0.1 to 0.5 the utilization of the third node decrease from 6773 packets to 3870 packets, while utilization of the first and second nodes remains constant when other parameters remain fixed.

It is also noticed that as the transmission rate of node 1(μ_1) increases, the utilization of the second and third node increase while the utilization of the first node decreases when other parameters remain fixed. As the transmission rate of node 2 (μ_2) increases the utilization of the second node decreases but for the utilization of the first and third nodes remain constant when other parameters remains fixed. As the transmission rate of node 3 (μ_3) increases, the utilization of the third node decreases but for the utilization of the first and second nodes remains constant when other parameters remain fixed.

Table 2: Values of Mean Number of Packets and Utilization

t	**a**	**b**	**α**	**β**	**μ_1**	**μ_2**	**μ_3**	**π**	**(1-π-δ)**	**δ**	**$L_1(t)$**	**$L_2(t)$**	**$L_3(t)$**	**$L_n(t)$**	**$U_1(t)$**	**$U_2(t)$**	**$U_3(t)$**
0.2	5	25	4	2	4	10	20	0.3	0.3	0.4	8.7276	0.6398	0.4167	9.7841	0.5679	0.343	0.2715
0.5	5	25	4	2	4	10	20	0.3	0.3	0.4	15.0987	1.5874	0.8568	17.5429	0.8864	0.6609	0.4948
0.8	5	25	4	2	4	10	20	0.3	0.3	0.4	18.59	2.0984	1.0849	21.7733	0.9633	0.7741	0.5861
1.2	5	25	4	2	4	10	20	0.3	0.3	0.4	22.017	2.5434	1.2969	25.8573	0.9864	0.84	0.6544
2.0	5	25	4	2	4	10	20	0.3	0.3	0.4	28.1206	3.2841	1.6647	33.0694	0.9966	0.9074	0.7433
0.5	**1**	25	4	2	4	10	20	0.3	0.3	0.4	13.0856	1.3758	0.7426	15.2039	0.873	0.6128	0.4488
0.5	**2**	25	4	2	4	10	20	0.3	0.3	0.4	13.5889	1.4287	0.7711	15.7886	0.8785	0.6261	0.4611
0.5	**3**	25	4	2	4	10	20	0.3	0.3	0.4	14.0921	1.4816	0.7997	16.3734	0.882	0.6386	0.4728
0.5	**4**	25	4	2	4	10	20	0.3	0.3	0.4	14.5954	1.5345	0.8282	16.9581	0.8845	0.6501	0.484
0.5	5	**10**	4	2	4	10	20	0.3	0.3	0.4	7.5494	0.7937	0.4284	807715	0.8722	0.4869	0.3214
0.5	5	**15**	4	2	4	10	20	0.3	0.3	0.4	10.0658	1.0583	0.5712	11.6953	0.8799	0.5631	0.3906
0.5	5	**20**	4	2	4	10	20	0.3	0.3	0.4	12.5823	1.3229	0.714	14.6191	0.884	0.6139	04475
0.5	5	**25**	4	2	4	10	20	0.3	0.3	0.4	15.0987	1.5874	0.8568	17.5429	0.8864	0.6609	0.4908
0.5	5	**30**	4	2	4	10	20	0.3	0.3	0.4	17.6152	1.852	0.9996	20.4668	0.8879	0.6934	0.5345
0.5	5	25	**3.0**	2	4	10	20	0.3	0.3	0.4	11.8562	1.2369	0.6698	13.763	0.816	0.5692	04134
0.5	5	25	**3.5**	2	4	10	20	0.3	0.3	0.4	13.4775	1.4122	0.7633	15.653	0.8554	0.618	0.4556
0.5	5	25	**4.5**	2	4	10	20	0.3	0.3	0.4	16.72	1.7627	0.9503	19.4320	0.9107	0.6992	0.5312
0.5	5	25	**5.0**	2	4	10	20	0.3	0.3	0.4	18.3412	1.9379	1.0437	21.3229	0.9298	0.7331	0.5649
0.5	5	25	4	**1.5**	4	10	20	0.3	0.3	0.4	14.5665	1.5411	0.8295	16.9372	0.8751	0.6503	0.4839
0.5	5	25	4	**2.5**	4	10	20	0.3	0.3	0.4	15.6309	1.6338	0.884	18.1487	0.8932	0.6713	0.5054
0.5	5	25	4	**3.0**	4	10	20	0.3	0.3	0.4	16.1631	1.6801	0.9113	18.7545	0.8916	0.6813	0.5158
0.5	5	25	4	**3.5**	4	10	20	0.3	0.3	0.4	16.6913	1.7264	0.9386	19.3603	0.9056	0.691	0.526
0.5	5	25	4	2	**3**	10	20	0.3	0.3	0.4	17.9478	1.3868	0.7556	20.0903	0.8918	0.6293	0.4636
0.5	5	25	4	2	**4**	10	20	0.3	0.3	0.4	15.0987	1.874	0.8568	17.5429	0.8864	06609	0.4948
0.	5	2	4	2	**5**	1	2	0.	0.	0.	12.913	1.725	0.923	15.6253	0.876	0.677	0.510

5		5				0	0	3	3	4	5	6	9		5	6	9
0.5	5	25	4	2	**6**	10	20	0.3	0.3	0.4	11.2130	1.8219	0.9688	14.001	0.8615	0.6862	0.5184
0.5	5	25	4	2	4	**8**	20	0.3	0.3	0.4	15.0987	1.8964	0.8568	17.8519	0.8864	0.7028	0.4948
0.5	5	25	4	2	4	**9**	20	0.3	0.3	0.4	15.0987	1.7294	0.8568	17.6849	0.8864	0.6816	0.4948
0.5	5	25	4	2	4	**11**	20	0.3	0.3	0.4	15.0987	1.4657	0.8568	17.4212	0.8864	0.6408	0.4948
0.5	5	25	4	2	4	**12**	20	0.3	0.3	0.4	15.0987	1.3603	0.8568	17.3519	0.8864	0.6215	0.4948
0.5	5	25	4	2	4	10	**8**	0.3	0.3	0.4	15.0987	1.5874	1.8964	1805825	0.8864	0.6609	0.7028
0.5	5	25	4	2	4	10	**12**	0.3	0.3	0.4	15.0987	1.5874	1.3603	18.0465	0.8864	0.6609	0.6215
0.5	5	25	4	2	4	10	**16**	0.3	0.3	0.4	15.0987	1.5874	1.0563	17.7391	0.8864	0.6609	0552
0.5	5	25	4	2	4	10	**22**	0.3	0.3	0.4	15.0987	1.5874	0.7834	17.4691	0.8864	0.6609	0.4701
0.5	5	25	4	2	4	10	**25**	0.3	0.3	0.4	15.0987	1.5874	0.6941	17.3802	0.8864	0.6609	0.437
0.5	5	25	4	2	4	10	20	**0.3**	**0.6**	**0.1**	15.0987	1.5874	1.7316	18.3997	0.8864	0.6609	0.6773
0.5	5	25	4	2	4	10	20	**0.3**	**0.5**	**0.2**	15.0987	1.5874	1.428	18.1141	0.8864	0.6609	0.6297
0.5	5	25	4	2	4	10	20	**0.3**	**0.4**	**0.3**	15.0987	1.5874	1.1424	17.8285	0.8864	0.6609	0.5723
0.5	5	25	4	2	4	10	20	**0.3**	**0.3**	**0.4**	15.0987	1.5874	0.8568	17.5429	0.8864	0.6609	0.4948
0.5	5	25	4	2	4	10	20	**0.3**	**0.2**	**0.5**	15.0987	1.5874	0.5712	17.2573	0.8864	0.6609	0.387

***= seconds, # = Multiple of 10,000 messages/second $ = Multiple of 10,000 packets/second**

From the equations (32), (39), (46) and (33), (40), (47)the throughput and average delay of the network are computed for different values of t, a, b, α, β, μ_1, μ_2, μ_3, π and $\delta$are given in Table 3.

It is observed that as time (t) varies from 0.2 to 2.0 seconds, the throughput of the first, second and third nodes increase from 22717 packets to 39865 packets, 34928 packets to 90743 packets and 54301 packets to 149068packets respectively when other parameters remain fixed. As the batch size distribution parameter (a) varies from 1 to 4, the throughput first, second and third node increase from 3492 packets to 3581 packets, 61276 packets to 65013 packets and 89765 packets to 96802 packets when other parameters are fixed at (0.5, 25, 4, 2, 10, 20, 0.3, 0.3, and 0.4) for (t, b, α, β, μ_1, μ_2, μ_3, π, $1-\pi-\delta$, δ). When batch size distribution parameter (b) varies from 10 to 30, the throughput of the first, second and third nodes increase from 34888 packets to 35517 packets, 48693 packets to 69338 packets and 64280 packets to 106895 packets when other parameters are fixed at (0.5, 5, 4, 2, 10, 20, 0.3, 0.3, and 0.4) for (t, a, α, β, μ_1, μ_2, μ_3, π, $1-\pi-\delta$, δ).

. As the arrival rate parameter (α) varies from $3.0\text{x}10^4$ messages/second to $5.0\text{x}10^4$ messages/second the throughput of the first, second, and third nodes increase from 32640 packets to 37193 packets, 56920 packets to 73313 packets and 82670 to 112982 packets respectively, when other parameters remain fixed. As the arrival rate parameters (β) varies $1.5\text{x}10^4$ messages/second to $3.5\text{x}10^4$ messages/second the throughput of the first, second, and third nodes increase from 35166 packets to 36223 packets, 65028 packets to 69098 packets and 96785 packets to 105198 packets respectively, when the other parameters are fixed at (0.5, 5, 25, 4, 10, 20, 0.4, 0.6) for (t, a, b, μ_1, μ_2, μ_3, π, δ).

When the transmission rate of node 1(μ_1) varies from $3\text{x}10^4$ packets/sec to $6 \text{ x } 10^4$ packets/sec, the throughput of the first, second and third nodes increase from 26754 packets to 43823 packets, 62927 packets to 68623 packets and 92728 packets to 103674 packets respectively, when other parameters remain fixed.

When the transmission rate of node 2(μ_2) varies from 8x10^4 packets/sec to 12 x 10^4 packets/sec, the throughput of the second nodes increase from 56224 packets to 98956 packets respectively and the first and third nodes remain constant when other parameters remain fixed. When the transmission rate of node 3(μ_3) varies from 12x10^4 packets/sec to 22 x 10^4 packets/sec, the throughput of the third node increases from 56224 packets to 10344 packets respectively and the first and second nodes remain constant when other parameters remain fixed. When the leaving rate parameter δ varies from 0.1 to 0.5 then the throughput of the third node increase from 134668 packets to 77735 packets respectively, but in the first and second buffers it remain constant when other parameters remain fixed.

Table 3: Values of Throughput and Mean Delay

t	a	b	α	B	μ_1	μ_2	μ_3	π	$(1-\pi-\delta)$	δ	$Thp_1(t)$	$Thp_2(t)$	$Thp_3(t)$	$W_1(t)$	$W_2(t)$	$W_3(t)$
0.2	5	25	4	2	4	10	20	0.3	0.3	0.4	2.2717	3.4298	5.4301	3.8419	1.865	0.0767
0.5	5	25	4	2	4	10	20	0.3	0.3	0.4	3.5455	6.6093	9.8956	4.2586	0.2402	0.0866
0.8	5	25	4	2	4	10	20	0.3	0.3	0.4	3.8533	7.7408	11.7229	4.8244	0.2711	0.0925
1.2	5	25	4	2	4	10	20	0.3	0.3	0.4	3.9457	8.3998	13.0884	5.58	0.3028	0.0991
2.0	5	25	4	2	4	10	20	0.3	0.3	0.4	3.9865	9.0743	14.9068	7.054	0.3619	0.1117
0.5	**1**	25	4	2	4	10	20	0.3	0.3	0.4	3.492	6.1276	8.9765	3.7472	0.2245	0.0827
0.5	**2**	25	4	2	4	10	20	0.3	0.3	0.4	3.5139	6.2612	9.2212	3.8671	0.2282	0.0836
0.5	**3**	25	4	2	4	10	20	0.3	0.3	0.4	3.5281	6.3855	9.455	3.9943	0.232	0.0846
0.5	**4**	25	4	2	4	10	20	0.3	0.3	0.4	3.5381	6.5013	9.6802	4.1251	0.236	0.0856
0.5	5	**10**	4	2	4	10	20	0.3	0.3	0.4	3.4888	4.8693	6.428	2.1639	0.163	0.0666
0.5	5	**15**	4	2	4	10	20	0.3	0.3	0.4	3.5197	5.6308	7.8126	2.8958	0.1879	0.0731
0.5	5	**20**	4	2	4	10	20	0.3	0.3	0.4	3.5359	6.1888	8.9502	3.5585	0.2137	0.0798
0.5	5	**25**	4	2	4	10	20	0.3	0.3	0.4	3.5455	6.6093	9.8956	4.2586	0.2402	0.0866
0.5	5	**30**	4	2	4	10	20	0.3	0.3	0.4	3.5517	6.9338	10.6895	4.9596	0.2671	0.0935
0.5	5	25	**3.0**	2	4	10	20	0.3	0.3	0.4	3.264	5.692	8.267	3.6324	0.2173	0.081
0.5	5	25	**3.5**	2	4	10	20	0.3	0.3	0.4	3.4216	6.1781	9.117	3.9389	0.2286	0.0838
0.5	5	25	**4.5**	2	4	10	20	0.3	0.3	0.4	3.6428	6.9919	10.6231	4.5898	0.2521	0.0895
0.5	5	25	**5.0**	2	4	10	20	0.3	0.3	0.4	3.7193	7.3313	11.2982	4.9313	0.2643	0.0934
0.5	5	25	4	**1.5**	4	10	20	0.3	0.3	0.4	3.5166	6.5028	9.6785	4.1423	0.237	0.0857
0.5	5	25	4	**2.5**	4	10	20	0.3	0.3	0.4	3.5727	6.7126	10.1081	4.3751	0.2434	0.0875
0.5	5	25	4	**3.0**	4	10	20	0.3	0.3	0.4	3.5983	6.8127	10.3162	4.491	0.2466	0.0883
0.5	5	25	4	**3.5**	4	10	20	0.3	0.3	0.4	3.6223	6.9098	10.5198	4.609	0.2499	0.0892
0.5	5	25	4	2	**3**	10	20	0.3	0.3	0.4	2.6754	6.2927	9.2728	6.7085	0.2204	0.0815
0.5	5	25	4	2	**4**	10	20	0.3	0.3	0.4	3.5455	6.6093	9.8956	4.2586	0.2402	0.0866
0.5	5	25	4	2	**5**	10	20	0.3	0.3	0.4	4.3823	6.7765	10.2172	2.9467	0.2546	0.0904
0.5	5	25	4	2	**6**	10	20	0.3	0.3	0.4	50169	6.8623	10.3674	2.1687	0.2655	0.0935
0.5	5	25	4	2	4	**8**	20	0.3	0.3	0.4	3.5455	5.6224	9.8956	4.2586	0.3373	0.0866
0.5	5	25	4	2	4	**9**	20	0.3	0.3	0.4	3.5455	6.1348	9.8956	4.2586	0.2819	0.0866
0.5	5	25	4	2	4	**11**	20	0.3	0.3	0.4	3.5455	7.0493	9.8956	4.2586	0.2079	0.0866
0.5	5	25	4	2	4	**12**	20	0.3	0.3	0.4	3.5455	7.4578	9.8956	4.2586	0.1824	0.0866
0.5	5	25	4	2	4	10	**8**	0.3	0.3	0.4	3.5455	6.6093	506224	4.2586	0.2402	0.3373
0.5	5	25	4	2	4	10	**12**	0.3	0.3	0.4	3.5455	6.6093	704578	4.2586	0.2402	0.1824
0.5	5	25	4	2	4	10	**16**	0.3	0.3	0.4	3.5455	6.6093	808328	4.2586	0.2402	0.1192
0.5	5	25	4	2	4	10	**22**	0.3	0.3	0.4	3.5455	6.6093	10.3411	4.2586	0.2402	0.0758
0.5	5	25	4	2	4	10	**25**	0.3	0.3	0.4	3.5455	6.6093	10.926	4.2586	0.2402	0.0635
0.5	5	25	4	2	4	10	20	**0.3**	**0.6**	**0.1**	3.5455	6.6093	13.4668	4.2586	0.2402	0.1272
0.5	5	25	4	2	4	10	20	**0.3**	**0.5**	**0.2**	3.5455	6.6093	12.5949	4.2586	0.2402	0.1134
0.5	5	25	4	2	4	10	20	**0.3**	**0.4**	**0.3**	3.5455	6.6093	11.4465	4.2586	0.2402	0.0998
0.5	5	25	4	2	4	10	20	**0.3**	**0.3**	**0.4**	3.5455	6.6093	9.8956	4.2586	0.2402	0.0866
0.5	5	25	4	2	4	10	20	**0.3**	**0.2**	**0.5**	3.5455	6.6093	7.7735	4.2586	0.2402	0.0738

*** = seconds, # = Multiple of 10,000 messages/second, $ = Multiple of 10,000 packets/second**

From Table 3, It is also observed that as time (t) varies from 0.2 to 2.0 seconds, the mean delay in the first and second and third transmitters increase from 384.19 μs to 705.4 μs, 18.65μs to 36.19 μs and 7.67 μs to 11.17 μs respectively, when other parameters remain fixed. As the batch size distribution parameter (a) varies from 1 to 5, the mean delay in the first, second and third transmitters increase from 374.72 μs to 412.52 μs, 22.45 μs to 23.63 μs and 8.56 μs respectively when other parameters are fixed. As the batch size distribution parameter (b) varies from10 to 30 , the mean delay in first , second and third transmitters and increase from 216.39 μs to 495.6 μs, 16.3 μs to 26.71 μs and 6.66μs to 9.35 μs respectively when other parameters are fixed. As the arrival rate parameter (α) varies from $3.0x10^4$ messages/second to $5.0x10^4$ messages/second, the mean delay in first, second and third transmitters increase 363.24 μs to 414.23 μs, 21.73 μs to 24.63 μs and 8.1 μs to 9.24 μs when other parameters are fixed.

As the arrival rate parameter (β) varies from $1.5x10^4$ messages/second to $3.5x10^4$ messages/second the mean delay in first, second and third transmitters increase 441.2 μs to 460.9 μs, 23.7 μs to 24.99μs and 9.24 μs to 8.92 μs when other parameters are fixed. As the transmission rate of node 1 (μ_1) varies form $3.0x10^4$ packets/second to $6.0x10^4$ packets /second, the mean delay in the first, second and third transmitters increase from670.85 μs to 294.67μs, 22.04 μs to 26.55 μs and 8.66 μs to 9.35 μs respectively when other parameters remain fixed. As the transmission rate of node 2 (μ_2) varies form $8.0x10^4$ packets/second to $12.0x10^4$ packets /second, the mean delay in the second transmitter decrease from 33.73μs to 20.79 μs, but in the first and third transmitters it remain constant at 425.86 μs and 8.66 μs when other parameters remain fixed. As the transmission rate of node 3 (μ_3) varies form 12.0 $x10^4$ packets/second to $22.0x10^4$ packets /second, the mean delay in the third transmitter decreases from 33.73 μs to 7.58μs, but in the first and second transmitters it remain constant at 425.86 μs and 24.02 μs when other parameters remain fixed.

When the leaving rate parameter (δ) varies from 0.1 to 0.5, the mean delay in third at transmitter decreases from 12.72 μs to 7.38 μs and the mean delay in first and second transmitters remain constant when other parameters remain fixed. The variance of the number of packets in each buffer, the coefficient of variation of the number of packets in first, second and third buffers are computed and given in Table 4.

It is observed that the dynamic bandwidth allocation strategy has significant influence on all performance measures of the network. It is further observed that the performance measures are highly sensitive towards smaller values of time. It is optimal to consider dynamic bandwidth allocation and evaluate the performance under transient conditions. It is also to be observed that the congestion in buffers and delays in transmission can be reduced to a minimum level by adopting dynamic bandwidth allocation. This is phenomenon has a vital bearing on quality of transmission (service).

Table 4: Values of Variance and Coefficient of Variation of the Number of Packets

t	a	b	α	β	μ_1	μ_2	μ_3	π	(1-π-δ)	δ	$V_1(t)$	$V_2(t)$	$V_3(t)$	$CV_1(t)$	$CV_2(t)$	$CV_3(t)$
0.2	5	25	4	2	4	10	20	0.3	0.3	0.4	113.3416	1.2405	0.6606	121.9837	174.0848	195.0484
0.5	5	25	4	2	4	10	20	0.3	0.3	0.4	159.4393	2.937	1.2719	83.6291	107.9588	131.6304
0.8	5	25	4	2	4	10	20	0.3	0.3	0.4	183.356	3.7105	1.5663	72.8398	91.797	115.362
1.2	5	25	4	2	4	10	20	0.3	0.3	0.4	211.6342	4.4165	1.8534	66.0747	82.6273	104.9711
2.0	5	25	4	2	4	10	20	0.3	0.3	0.4	267.0789	5.665	2.3692	58.1159	72.4735	92.4642
0.5	**2**	25	4	2	4	10	20	0.3	0.3	0.4	140.3745	2.6141	1.1358	87.1889	113.1687	138.2056
0.5	**3**	25	4	2	4	10	20	0.3	0.3	0.4	146.3393	2.7181	1.18	85.8427	111.2761	135.842
0.5	**4**	25	4	2	4	10	20	0.3	0.3	0.4	152.6943	2.8257	1.2254	84.6631	109.5455	133.6565
0.5	5	**10**	4	2	4	10	20	0.3	0.3	0.4	37.7829	1.0764	0.5153	81.4211	130.7139	167.5739
0.5	5	**15**	4	2	4	10	20	0.3	0.3	0.4	68.5823	1.6054	0.7395	82.2729	119.7265	150.5509
0.5	5	**20**	4	2	4	10	20	0.3	0.3	0.4	109.1344	2.2256	0.9917	83.0275	112.7748	139.474
0.5	5	**25**	4	2	4	10	20	0.3	0.3	0.4	059.4393	2.937	1.2719	83.6291	107.9588	131.6304
0.5	5	**30**	4	2	4	10	20	0.3	0.3	0.4	219.4969	3.7396	1.5802	84.106	104.4168	125.7583
0.5	5	25	**3.0**	2	4	10	20	0.3	0.3	0.4	125.9282	2.2907	0.9957	94.6487	122.3611	148.9627
0.5	5	25	**3.5**	2	4	10	20	0.3	0.3	0.4	142.6838	2.6138	1.1338	88.6296	114.4862	139.4954
0.5	5	25	**4.5**	2	4	10	20	0.3	0.3	0.4	176.1949	3.2602	1.4101	79.3891	102.4341	124.9614
0.5	5	25	**5.0**	2	4	10	20	0.3	0.3	0.4	192.9504	3.5833	1.548	75.7347	97.6791	119.213
0.5	5	25	4	**1.5**	4	10	20	0.3	0.3	0.4	153.0906	2.8491	1.2302	84.9411	109.5279	133.707
0.5	5	25	4	**2.5**	4	10	20	0.3	0.3	0.4	165.788	3.0249	1.3136	82.3744	106.4252	129.647
0.5	5	25	4	**3.0**	4	10	20	0.3	0.3	0.4	172.1368	3.1128	1.3554	81.1731	105.0127	127.7506
0.5	5	25	4	**3.5**	4	10	20	0.3	0.3	0.4	178.4855	3.2007	1.3971	80.0217	103.6273	125.9349
0.5	5	25	4	2	**3**	10	20	0.3	0.3	0.4	202.2947	2.3967	1.0625	79.2456	111.6328	136.4137
0.5	5	25	4	2	**4**	10	20	0.3	0.3	0.4	159.4393	2.937	1.2719	83.6295	107.9588	131.6304
0.5	5	25	4	2	**5**	10	20	0.3	0.3	0.4	130.6819	3.3645	1.4359	88.5247	106.2993	129.7054
0.5	5	25	4	2	**6**	10	20	0.3	0.3	0.4	110.3668	3.7083	1.5684	93.7136	105.6957	129.2635
0.5	5	25	4	2	4	**8**	20	0.3	0.3	0.4	159.4393	3.8011	1.2719	83.6291	102.8089	131.6304
0.5	5	25	4	2	4	**9**	20	0.3	0.3	0.4	159.4393	3.3217	1.2719	83.6291	105.3877	131.6304
0.5	5	25	4	2	4	**11**	20	0.3	0.3	0.4	159.4393	3.6233	1.2719	83.6291	110.5068	131.6304
0.5	5	25	4	2	4	**12**	20	0.3	0.3	0.4	159.4393	2.3638	1.2719	83.6291	113.0218	131.6304
0.5	5	25	4	2	4	10	**8**	0.3	0.3	0.4	159.4393	2.937	3.8011	83.6291	107.9588	102.8089
0.5	5	25	4	2	4	10	**12**	0.3	0.3	0.4	159.4393	2.937	2.3638	83.6291	107.9588	113.0218
0.5	5	25	4	2	4	10	**16**	0.3	0.3	0.4	159.4393	2.937	1.6683	83.6291	107.9588	122.6676
0.5	5	25	4	2	4	10	**22**	0.3	0.3	0.4	159.4393	2.937	1.1332	83.6291	107.9588	135.8792
0.5	5	25	4	2	4	10	20	**0.3**	**0.6**	**0.1**	159.4393	2.937	3.3741	83.6291	107.9588	107.1952
0.5	5	25	4	2	4	10	20	**0.3**	**0.5**	**0.2**	159.4393	2.937	2.5811	83.6291	107.9588	112.5076
0.5	5	25	4	2	4	10	20	**0.3**	**0.4**	**0.3**	159.4393	2.937	1.8804	83.6291	107.9588	120.0362
0.5	5	25	4	2	4	10	20	**0.3**	**0.3**	**0.4**	159.4393	2.937	1.2719	83.6291	107.9588	131.6304
0.5	5	25	4	2	4	10	20	**0.3**	**0.2**	**0.5**	159.4393	2.937	0.7557	83.6291	107.9588	152.1916

*** = seconds, # = Multiple of 10,000 messages/second, $ = Multiple of 10,000 packets/second**

6. Sensitivity Analysis of the Model when Batch Size Distribution Is Uniform Distribution

Sensitivity analysis of the Model is Performed with respect to the parameters t, a, b, α, β, μ_1, μ_2, μ_3, π and θ on the mean number packets in the first and second buffers, mean delay in the first , second and Third transmitter, utilization and throughput of first, second and third nodes.

The following data has been considered for the sensitivity analysis.
t = 0.5 sec, a = 5, b = 25, α = $4\text{x}10^4$ messages/second, μ_1 = $4\text{x}10^4$ packets/second,
μ_2 = $10\text{x}10^4$ packets/second, μ_3 = $20\text{x}10^4$ messages/second, δ = 0.4 and π = 0.3

The performance measures of the model are computed with variation of -15%, -10%, 0%, +5%, +10% and +15% on the input parameters t, a, b, α, β, μ_1, μ_2, μ_3, π and θ and -60%, -40%, -20%, 0%, +20%, +40% and +60% on the batch size distribution parameters a and b to retain them as integers . It is observed that the performance measures are highly affected by time (t) and the batch size distribution.

As (t) increases to 15% the average number of packets in the three buffers increase along with the average delays in buffers. Similarly, as arrival rate of messages increases by 15%, the average number of packets in the three buffers increase along with the average delays in buffers. Overall analysis of the parameters reflects that the dynamic bandwidth allocation strategy for congestion control tremendously reduces the delays in communication and improves voice quality by reducing burstness in buffers.

7. Comparative Study

To study the effect of non-homogenous Poisson arrivals on the communication network a comparative study between the performance measure of the network models with non-homogenous Poisson arrivals and Poisson arrivals is performed. The performance measure of both models is computed with fixed values of the parameters (a, b, α, β, μ_1,μ_2,μ_2 π, θ) and different values of t=0.2, 0.5, 0.8, 1.2, 2.0 seconds and presented in Table 5.

As t increases the percentage variation of performance measure between the models is increasing. For the model with non-homogenous Poisson arrivals with dynamic bandwidth allocation has more utilization compared to that of the model with Poisson arrivals with dynamic bandwidth allocation. From this analysis it is observed that the assumption of non-homogenous Poisson arrivals have significant influence on all the performance measure of the network.

Table 5 Comparative study of models with non-homogenous compound Poisson and homogenous Poisson arrivals when batch sized distribution is uniform

Time (t) Sec	Parameters Measured	Model With Nhp β =2	Model With Hp β =0	Difference	% Variation
t=0.2	L1(t)	8.7276	8.2601	0.4675	2.7519
	L2(t)	0.6398	0.6144	0.0254	2.0251
	L3(t)	0.4167	0.3986	0.0181	2.2200
	U1(t)	0.5679	0.5503	0.0176	1.5739
	U2(t)	0.343	0.3315	0.0115	1.7049
	U3(t)	0.2715	0.2613	0.0102	1.9144
	W1(t)	3.8419	3.7525	0.0894	1.1771
	W2(t)	0.1865	0.1854	0.0011	0.2957
	W3(t)	0.0767	0.0763	0.0004	0.2614
	Thp1(t)	2.2717	2.2012	0.0705	1.5761
	Thp2(t)	3.4298	3.3148	0.115	1.7050
	Thp3(t)	5.4301	5.2263	0.2038	1.9124
	Thp3(t)	11.7229	10.0542	1.6687	7.6626
t=1.2	L1(t)	22.017	14.8766	7.1404	19.3546
	L2(t)	2.5434	1.7753	0.7681	17.7854
	L3(t)	1.2969	0.8907	0.4062	18.5682
	U1(t)	0.9864	0.9554	0.031	1.5964
	U2(t)	0.84	0.7254	0.1146	7.3208
	U3(t)	0.6544	0.5203	0.1341	11.4156
	W1(t)	5.58	3.8929	1.6871	17.8097
	W2(t)	0.3028	0.2447	0.0581	10.6118
	W3(t)	0.0991	0.0856	0.0135	7.3091
	Thp1(t)	3.9457	3.8214	0.1243	1.6003
	Thp2(t)	8.3998	7.2541	1.1457	7.3189
	Thp3(t)	13.0884	10.4064	2.682	11.4152

8. Conclusion

This paper introduces the designing and analysis of forked communication network model with non-homogenous bulk arrivals having intermediate departures. Here it is assumed that two nodes are in parallel and connected to the first node in tandem. Packet arrivals to the first buffer are characterised by non-homogenous compound Poisson process.

The non-homogenous compound Poisson process can portray the arrivals of the traffic more effectively science it includes stationery and non-stationary traffic along with single are bulk arrivals of packets. The non-stationary traffic i.e. time dependent nature is to be included in modelling the communication networks in order to improve the performance of the network. After getting transmission from the first node the packets may join the second or third buffers or leave the network with certain probabilities. The behaviours of the proposed network is analysed by deriving the explicit expression for the performance measures such as mean content of the buffers, the throughput of the nodes, the utilization of the transmitters, the mean delay in transmission. The sensitivity analysis of the model revealed that the dynamic bandwidth allocation can reduce the burstness in buffers and improve the quality of service. It is also observed that the time dependent analysis can be predict the performance measures more close to the reality. It is further observed that the bulk size distribution parameters are also influencing the performance. The comparative study states that the proposed network model is a versatile model since it includes several of the earlier models as particular cases. The network managers can optimally predicted the performance of LAN, WAN, MAN by estimating the model performance with the historical data. It is also possible to consider the time dependent transmission rates for the proposed network which will be studied separately.

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